1. Freezy Co. has three factories *A*, *B* and *C*. It supplies freezers to three shops *D*, *E* and *F*. The table shows the transportation cost in pounds of moving one freezer from each factory to each outlet. It also shows the number of freezers available for delivery at each factory and the number of freezers required at each shop. The total number of freezers required is equal to the total number of freezers available.

	D	*E*	*F*	Available
A	21	24	16	24
B	18	23	17	32
C	15	19	25	14
Required	20	30	20	

(a) Use the north-west corner rule to find an initial solution.

You may not wish to use all of these tables.

	D	*E*	*F*
A	21	24	16
B	18	23	17
C	15	19	25

	D	*E*	*F*
A	21	24	16
B	18	23	17
C	15	19	25

	D	*E*	*F*
A	21	24	16
B	18	23	17
C	15	19	25

	D	*E*	*F*
A	21	24	16
B	18	23	17
C	15	19	25

	D	*E*	*F*
A	21	24	16

B	18	23	17
C	15	19	25

(2)

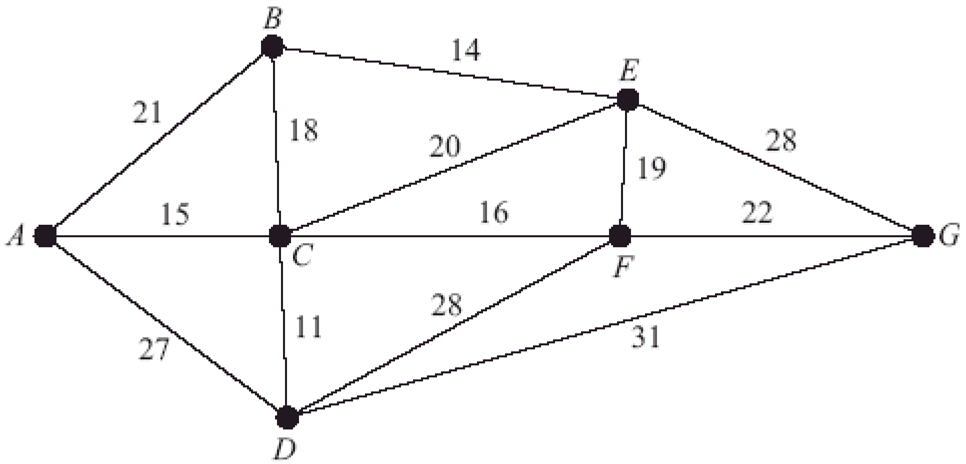
(b) Obtain improvement indices for each unused route.

(5)

(c) Use the stepping-stone method **once** to obtain a better solution and state its cost.

(4)
(Total 11 marks)

2.



The network in the figure above shows the distances, in km, of the cables between seven electricity relay stations *A*, *B*, *C*, *D*, *E*, *F* and *G*. An inspector needs to visit each relay station. He wishes to travel a minimum distance, and his route must start and finish at the same station.

By deleting *C*, a lower bound for the length of the route is found to be 129 km.

(a) Find another lower bound for the length of the route by deleting *F*. State which is the best lower bound of the two.

(5)

(b) By inspection, complete the table of least distances below.

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	21	15	26	35	31	53
B	21	–	18	29	14		42
C	15	18	–	11	20	16	38
D	26	29	11	–		27	31
E	35	14	20		–	19	28
F	31		16	27	19	–	22
G	53	42	38	31	28	22	–

(2)

The table can now be taken to represent a complete network.

(c) Using the nearest-neighbour algorithm, starting at *F*, obtain an upper bound to the length of the route. State your route.

(4)
(Total 11 marks)

3. Three warehouses *W*, *X* and *Y* supply televisions to three supermarkets *J*, *K* and *L*. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

	J	*K*	*L*
W	3	6	3
X	5	8	4
Y	2	5	7

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 6 marks)

4. The table below shows the cost of transporting one block of staging from each of two supply points, *x* and Y, to each of four concert venues, A, B, C and D. It also shows the number of blocks held at each supply point and the number of blocks required at each concert venue. A minimal cost solution is required.

	A	B	C	D	Supply
X	28	20	19	16	53
Y	15	12	14	17	47
Demand	18	31	22	29	

(a) Use the north-west corner method to obtain a possible solution.

You may not need to use all of these tables.

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

	A	B	C	D
X				
Y				

(1)

(b) Taking the most negative improvement index to indicate the entering square, use the stepping stone method **twice** to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, routes, entering cells and exiting cells.

(9)

(c) Is your current solution optimal? Give a reason for your answer.

(1)
(Total 11 marks)

5. The table below shows the cost, in pounds, of transporting one unit of stock from each of three supply points, X, Y and Z to three demand points, A, B and C. It also shows the stock held at each supply point and the stock required at each demand point.

	A	**B**	**C**	Supply
X	17	8	7	22
Y	16	12	15	17
Z	6	10	9	15
Demand	16	15	23	

(a) This is a **balanced problem**. Explain what this means.

(1)

(b) Use the north west corner method to obtain a possible solution.

You may not need to use all of these tables.

	A	**B**	**C**
X			
Y			
Z			

	A	**B**	**C**
X			
Y			
Z			

	A	**B**	**C**
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	B	C
X			
Y			
Z			

	A	**B**	**C**
X			
Y			
Z			

	A	**B**	**C**
X			
Y			
Z			

(1)

(c) Taking ZA as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear and state your exiting cell.

(3)

(d) Perform one more iteration of the stepping-stone method to find a further improved solution. You must make your shadow costs, improvement indices, entering cell, exiting cell and route clear.

(6)

(e) State the cost of the solution you found in part (d).

(1)
(Total 12 marks)

6. Jameson cars are made in two factories A and B. Sales have been made at the two main showrooms in London and Edinburgh. Cars are to be transported from the factories to the showrooms. The table below shows the cost, in pounds, of transporting one car from each factory to each showroom.

It also shows the number of cars available at each factory and the number required at each showroom.

	London (L)	Edinburgh (E)	Supply
A	80	70	55
B	60	50	45
Demand	35	60	

It is decided to use the transportation algorithm to obtain a minimal cost solution.

(a) Explain why it is necessary to add a dummy demand point.

(2)

(b) Complete the table below.

	L	E	Dummy	Supply
A	80	70		55
B	60	50		45
Demand	35	60		100

(2)

(c) Use the north-west corner rule to obtain a possible pattern of distribution.

	L	E	D
A			
B			

(1)

(d) Taking the most negative improvement index to indicate the entering square, use the stepping-stone method to obtain an optimal solution. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal.

You may not need to use all of these tables

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

	L	E	D
A			
B			

(7)

(e) State the cost of your optimal solution.

(1)
(Total 13 marks)

7. A group of students and teachers from a performing arts college are attending the Glasenburgh drama festival. All of the group want to see an innovative modern production of the play 'The Decision is Final'. Unfortunately there are not enough seats left for them all to see the same performance.

There are three performances of the play, 1, 2, and 3. There are two types of ticket, Adult and Student. Student tickets will be purchased for the students and Adult tickets for the teachers.

The table below shows the price of tickets for each performance of the play.

	Adult	Student
Performance 1	£5.00	£4.50
Performance 2	£4.20	£3.80
Performance 3	£4.60	£4.00

There are 18 teachers and 200 students requiring tickets.
There are 94, 65 and 80 seats available for performances 1, 2, and 3 respectively.

(a) Complete the table below.

	Adult	Student	Dummy	Seats available
Performance 1	£5.00	£4.50		
Performance 2	£4.20	£3.80		
Performance 3	£4.60	£4.00		
Tickets needed				

(2)

(b) Explain why a dummy column was added to the table above.

(1)

(c) Use the north-west corner method to obtain a possible solution.

	Adult	Student	Dummy
1			
2			
3			

(1)

(d) Taking the most negative improvement index to indicate the entering square, use the stepping stone method **once** to obtain an improved solution. You must make your shadow costs and improvement indices clear.

	Adult	Student	Dummy
1			
2			
3			

	Adult	Student	Dummy
1			
2			

3			

	Adult	Student	Dummy
1			
2			
3			

	Adult	Student	Dummy
1			
2			
3			

	Adult	Student	Dummy
1			
2			
3			

	Adult	Student	Dummy
1			
2			
3			

(6)

After a further iteration the table becomes:

	Adult	Student	Dummy
Performance 1		73	21
Performance 2	18	47	
Performance 3		80	

(e) Demonstrate that this solution gives the minimum cost, and find its value.

(6)

(Total 16 marks)

8. (a) Explain briefly the circumstances under which a **degenerate** feasible solution may occur to a transportation problem.

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(2)

(b) Explain why a dummy location may be needed when solving a transportation problem.

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(1)

The table below shows the cost of transporting one unit of stock from each of three supply points *A*, *B* and *C* to each of two demand points 1 and 2. It also shows the stock held at each supply point and the stock required at each demand point.

	1	2	Supply
A	62	47	15
B	61	48	12
C	68	58	17
Demand	16	11	

(c) Complete the table below to show a possible initial feasible solution generated by the north-west corner method.

	1	2	3
A			
B			0
C			

(1)

(d) Use the stepping-stone method to obtain an optimal solution and state its cost. You should make your method clear by stating shadow costs, improvement indices, stepping-stone route, and the entering and exiting squares at each stage.

(10)
(Total 14 marks)

You may not wish to use all of these tables

	1	2	3
A			
B			0
C			

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	1	2	3
A			
B			0
C			

....

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....

....

	1	2	3
A			
B			0
C			

....

....

....

....

	1	2	3
A			
B			0
C			

....

....

....

....

	1	2	3
A			
B			0
C			

....

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....

	1	2	3
A			
B			0
C			

....

....

....

....

	1	2	3
A			
B			0
C			

....

....

....

....

	1	2	3
A			
B			0
C			

....

....

....

....

9. Three depots, F, G and H, supply petrol to three service stations, S, T and U. The table gives the cost, in pounds, of transporting 1000 litres of petrol from each depot to each service station.

	S	T	U
F	23	31	46
G	35	38	51
H	41	50	63

F, G and H have stocks of 540 000, 789 000 and 673 000 litres respectively.

S, T and U require 257 000, 348 000 and 412 000 litres respectively. The total cost of transporting the petrol is to be minimised.

Formulate this problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 8 marks)

10. The following minimising transportation problem is to be solved.

	J	K	Supply
A	12	15	9
B	8	17	13
C	4	9	12
Demand	9	11	

(a) Complete the table below.

	J	K	L	Supply
A	12	15		9
B	8	17		13
C	4	9		12
Demand	9	11		34

(2)

(b) Explain why an extra demand column was added to the table above.

(2)

A possible north-west corner solution is:

	J	K	L
A	9	0	
B		11	2
C			12

(c) Explain why it was necessary to place a zero in the first row of the second column.

(1)

After three iterations of the stepping-stone method the table becomes:

	J	K	L
A		8	1
B			13
C	9	3	

(d) Taking the most negative improvement index as the entering square for the stepping stone method, solve the transportation problem. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal.

(11)
(Total 16 marks)

11. Freezy Co. has three factories *A*, *B* and *C*. It supplies freezers to three shops *D*, *E* and *F*. The table shows the transportation cost in pounds of moving one freezer from each factory to each outlet. It also shows the number of freezers available for delivery at each factory and the number of freezers required at each shop. The total number of freezers required is equal to the total number of freezers available.

	D	*E*	*F*	Available
A	21	24	16	24
B	18	23	17	32
C	15	19	25	14
Required	20	30	20	

(a) Use the north-west corner rule to find an initial solution.

(2)

(b) Obtain improvement indices for each unused route.

(5)

(c) Use the stepping-stone method **once** to obtain a better solution and state its cost.

(4)
(Total 11 marks)

12. Three warehouses *W*, *X* and *Y* supply televisions to three supermarkets *J*, *K* and *L*. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

	J	*K*	*L*
W	3	6	3
X	5	8	4
Y	2	5	7

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 7 marks)

13. (a) Describe a practical problem that could be solved using the transportation algorithm.

(2)

A problem is to be solved using the transportation problem. The costs are shown in the table. The supply is from *A*, *B* and *C* and the demand is at *d* and *e*.

	d	*e*	Supply
A	5	3	45
B	4	6	35
C	2	4	40
Demand	50	60	

(b) Explain why it is necessary to add a third demand *f*.

(1)

(c) Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.

	d	*e*	*f*	Supply
A	5	3		45
B	4	6		35
C	2	4		40
Demand	50	60		

-

	d	*e*	*f*
A			
B			
C			

	d	*e*	*f*
A			
B			
C			

(5)

(d) Calculate shadow costs and improvement indices for this pattern.

	d	*e*	*f*
A			
B			
C			

(5)

(e) Use the stepping-stone method once to obtain an improved solution and its cost.

	d	*e*	*f*
A			
B			
C			

-

	d	*e*	*f*
A			
B			
C			

(5)
(Total 16 marks)

14. The manager of a car hire firm has to arrange to move cars from three garages *A*, *B* and *C* to three airports *D*, *E* and *F* so that customers can collect them. The table below shows the transportation cost of moving one car from each garage to each airport. It also shows the number of cars available in each garage and the number of cars required at each airport. The total number of cars available is equal to the total number required.

	Airport *D*	Airport *E*	Airport *F*	Cars available
Garage *A*	£20	£40	£10	6
Garage *B*	£20	£30	£40	5
Garage *C*	£10	£20	£30	8
Cars required	6	9	4	

(a) Use the North-West corner rule to obtain a possible pattern of distribution and find its cost.

	Airport *D*	Airport *E*	Airport *F*
Garage *A*			
Garage *B*			
Garage *C*			

(3)

(b) Calculate shadow costs for this pattern and hence obtain improvement indices for each route.

(4)

(c) Use the stepping-stone method to obtain an optimal solution and state its cost.

(7)
(Total 14 marks)

You may not need to use all of these grids

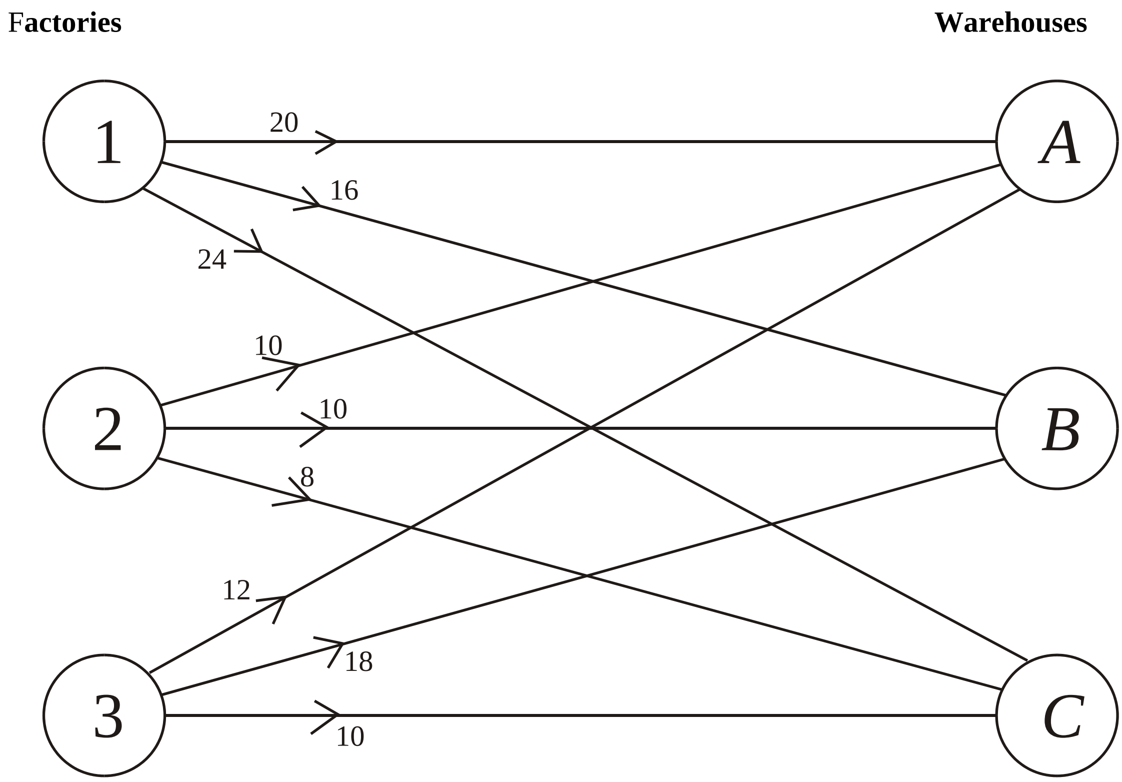
	Airport *D*	Airport *E*	Airport *F*
Garage *A*			
Garage *B*			
Garage *C*			

	Airport *D*	Airport *E*	Airport *F*
Garage *A*			
Garage *B*			
Garage *C*			

-

	Airport *D*	Airport *E*	Airport *F*
Garage *A*			
Garage *B*			
Garage *C*			

15.



A product is produced at three factories 1, 2 and 3 and shipped to three warehouses *A*, *B* and *C*. The transportation costs, in £ per unit, on the possible routes are shown in the diagram above. The capacities of the factories and the demands of the warehouses are shown in the tables below.

Factory	Capacity
1	300
2	500
3	100

Warehouse	Demand
A	200
B	400
C	300

Formulate, as a linear programming problem, the above situation when the total overall cost is to be minimised. Give reasons for your equations.

(Total 7 marks)

16. A clothing group owns a factory in each of three towns P, Q and S which distribute their products to three retail shops A, B and C. Factory availabilities, projected store demands and unit shipping costs are given in the table below.

To / **From**	***A***	***B***	***C***	**Factory Availability**
P	3	3	9	35
Q	6	7	6	60
S	5	2	8	30
Store Demand	20	35	70	

The group wishes to transport its products from factories to shops at minimum total cost.

(a) Write down the transportation pattern obtained by using the North-West corner rule.

(3)

(b) By calculating improvement indices I_{ij} , show that this pattern is not optimal.

(5)

(c) Use the stepping stone method to obtain an improved solution.

(3)

(d) Show that the transportation pattern obtained in part (*c*) is optimal and find the cost of this transportation pattern.

(6)
(Total 17 marks)

1. (a)

	D	*E*	*F*
A	20	4	
B		26	6
C			14

M1

A1 2

(b) $S_A = 0$ $S_B = -1$ $S_c = 7$

$D_D = 21$ $D_E = 24$ $D_F = 18$ M1

$I_{13} = I_{AF} = 16 - 0 - 18 = -2$ A1

$I_{21} = I_{BD} = 18 + 1 - 21 = -2$ M1

$I_{31} = I_{CD} = 15 - 7 - 21 = -13$ * A1 ft

$I_{32} = I_{CE} = 19 - 7 - 24 = -12$ A1 ft 5

(c) Eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-) \rightarrow = 14$ M1A1 ft

	D	*E*	*F*
A	6	18	
B		12	20
C	14		

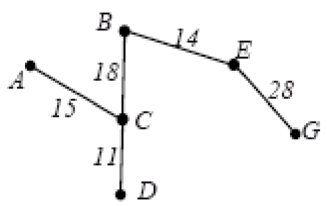
Cost £1384 A1 ft A1 4

[11]

2. (a) Deleting *F* leaves r.s.t M1

r.s.t length = 86 A1

So lower bound = 86 + 16 + 19 = 121



M1 A1 4

Best *LB* is 129 by deleting *C* B1ft 1

(b) Add 33 to *BF* and *FB* B1

Add 31 to *DE* and *ED* B1 2

(c) Tour, visits each vertex, order correct using table of least distances. M1 A1

e.g F C D A B E G F (actual route *F C D C A B E G F*) A1

Upper bound of 138 km A1 4

[11]

3. Let x_ij be number of units transported from i to j

Where $i \in \{W, X, Y\}$ and $j \in \{J, K, L]$

Warehouse Supermarket B1

Objective minimise "c" = $3x_{WJ} + 6x_{WK} + 3x_{WL} + 5x_{XJ} + 8x_{Xk} + 4x_{xl} + 2x_{YJ} + 5x_{Yk} + 7x_{Yl}$ B1

Subject to $x_{WJ} + x_{WK} + x_{WL} = 34$

$x_{XJ} + x_{XK} + x_{XL} = 57$ M1 A1

$x_{YJ} + x_{YK} + x_{YL} = 25$ A1

$x_{WJ} + x_{XJ} + x_{YJ} = 20$

$x_{WK} + x_{XK} + x_{YK} = 56$

$x_{WL} + x_{XL} + x_{YL} = 40$

$x_{ij} > 0 \quad \forall i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 6

[6]

4. (a)

	A	B	C	D
X	18	31	4	
Y			18	29

B1 1

Notes

1B1: Cao

(b) e.g.

		28	20	19	22
		A	B	C	D
0	X	x	x	x	–6
–5	Y	–8	–3	x	x

M1 A1

	A	B	C	D
X	18 – θ	31	4+θ	
Y	θ		18 – θ	29

2M1

Entering cell: AY $\theta = 18$

Either

Exiting cell: XA

	A	B	C	D
X		31	22	
Y	18		0	29

Or

Exiting cell: YC

	A	B	C	D
X	0	31	22	
Y	18			29

2A1ft

Either

		20	20	19	22
		A	B	C	D
0	X	8	x	x	–6
–5	Y	x	–3	x	x

Or

		28	20	19	30
		A	B	C	D
0	X	x	x	x	–14
–13	Y	x	5	8	x

3M1
3A1

Either

Exiting cell: XD

	A	B	C	D
X		31	22–θ	θ
Y	18		0+θ	29–θ

Exiting cell: XC
θ = 22

Or

Exiting cell: XD

	A	B	C	D
X	0–θ	31	22	
Y	18+θ			29–θ

Exiting cell: XA
θ = 0

4M1

Either

	A	B	C	D
X		31		22
Y	18		22	7

Or

	A	B	C	D
X		31	22	0
Y	18			29

4A1ft

Either

		14	20	13	16
		A	B	C	D
0	X	14	X	6	x
1	Y	x	–9	x	x

Or

		14	20	19	16
		A	B	C	D
0	X	14	x	x	x
1	Y	x	–9	–6	x

5A1

Notes

1M1: 6 shadow costs and precisely 3 improvement indices stated. (no extra zeros)
1A1: cao.
2M1: A valid route, negative II chosen, only one empty square used, θ's balance.
2A1ft: improved solution (no extra zeros)
3M1ft: 6 shadow costs and precisely 3 improvement indices stated (no extra zeros)
3A1: cao.
4M1ft:A valid route, negative II chosen, only one empty square used, θ's balance.
4A1ft: improved solution (no extra zeros)
5A1=5M1: 6 shadow costs and precisely 3 improvement indices, (or 1 negative improvement index), stated (no extra zeros).

Misreads - Not choosing most negative.

	A	B	C	D
X	18	31	4	
Y			18	29

		28	20	19	22
		A	B	C	D
0	X	x	x	x	–6
–5	Y	–8	–3	x	x

Either

Entering cell: XD

	A	B	C	D
X	18	31	4–θ	θ
Y			18+θ	29–θ

Exiting cell: XC
θ = 4

	A	B	C	D
X	18	31		4
Y			22	25

		28	20	13	16
		A	B	C	D
0	X	x	x	6	x
1	Y	–14	–9	x	x

Or

Entering cell: XD

	A	B	C	D
X	18	31–θ	4+θ	
Y		θ	18–θ	29

Exiting cell: YC
θ = 18

	A	B	C	D
X	18	13	22	
Y		18		29

		28	20	19	25
		A	B	C	D
0	X	x	x	x	–9
–8	Y	–5	x	3	x

Candidates can get
2M1 2A1 for first route and the improved solution

3M1 3A0 – 6 shadow costs and 3 IIs
4M1 for finding a valid route and 4A1 if their route leads to an improved solution
[A0 – 6 shadow costs and 3 IIs but it is CAO] 9

(c) Negative improvement index so not optimal B1ft 1

Notes

1B1ft=1A1ft: cao for conclusion, but must follow from at least one negative in a third 'set' of IIs.

[11]

5. (a) The supply is equal to the demand B1 1

(b)

	A	B	C
X	16	6	
Y		9	8
Z			15

B1 1

(c)

	A	B	C
X	16– θ	6+ θ	
Y		9– θ	8+ θ
Z	θ		15– θ`

M1 A1

Value of θ = 9, exiting cell is YB A1 3

(d)

		17	18	20
		A	B	C
0	X	7	15	
–5	Y			17
–11	Z	9		6

M1 A1

XC = 7 – 0 – 20 = –13

YA = 16 + 5 – 17 = 4

YB = 12 + 5 – 8 = 9

ZB = 10 + 11 – 8 = 13 A1 3

	A	B	C
X	$7-\theta$	15	θ
Y			17
Z	$9+\theta$		$6-\theta$

Value of θ = 6, entering cell XC, exiting cell ZC M1 A1

	A	B	C
X	1	15	6
Y			17
Z	15		

A1 3

Cost (£) 524 B1 1

[12]

6. (a) Total supply > total demand B2,1,0 2

(b) Adds 0, 0 and 5 to the dummy column B2,1,0 2

(c) B1 1

	L	E	D
A	35	20	
B		40	5

(d) M1A1

		80	70	20
		L	E	D
0	A	35	20	
–20	B		40	5

$I_{AD} = 0 - 0 - 20 = -20$
$I_{BL} = 60 + 20 - 80 = 0$ A1 3

M1

	L	E	D
A	35	$20 - \theta$	θ
B		$40+\theta$	$5-\theta$

$\theta = 5$; entering square is AD; exiting square is BD A1ft 2

B1ft

		80	70	0
		L	E	D
0	A	35	15	5
–20	B		45	

$I_{BL} = 60 + 20 - 80 = 0$
$I_{BD} = 0 + 20 - 0 = 20$ B1ft 2

(e) Cost is (£) 6100 B1 1

[13]

7. (a) B2,1,0 2

	A	S	D	Seats
1			0	94
2			0	65
3			0	80
	18	200	21	

(b) total supply > total demand B1 1

(c)(d)

	A	S	D	
1	18	76		B1
2		65		M1A1ft
3		59	21	

S(1) = 0 D(A) = 5
S(2) = –0.7 D(S) = 4.5
S(3) = –0.5 D(D) = 0.5

I_{1D} = 0 – 0 – 0.5 = –0.5 *
I_{2A} = 4.2 + 0.7 – 5 = –0.1
I_{2D} = 0 + 0.7 – 0.5 = 0.2
I_{3A} = 4.6 + 0.5 – 5 = 0.1 A1

	A	S	D			A	S	D		
1	18	76 – θ	θ	Entering 1D	1	18	55	21	M1A1ft	
2		65		Exiting 3D	2		65		A1	7
3		59 + θ	21 – θ	θ = 21	3		80			

(e) S(1) = 0 D(A) = 4.9 M1
S(2) = –0.7 D(B) = 4.5 A1
S(3) = –0.5 D(B) = 0

I_{1A}= 5 – 0 – 4.9 = 0.1
I_{2D} = 0 + 0.7 – 0 = 0.7
I_{3A} = 4.6 + 0.5 – 4.9 = 0.2
I_{3D} = 0 + 0.5 – 0 = 0.5 A1

Optimal since all II's ≥ 0 A1
cost £902.70 M1A1 6

[16]

8. (a) Either e.g.

In an n × m problem, a degenerate solution occurs when the number of cells used is less than (n + m – 1) B2,1,0 2

or e.g. when all the demand for one destination is satisfied by all the supply from a source, before the final demand and supplies are allocated

B2 cao
B1 cloze "bod" is B1

(b) If the total supply > total demand a dummy is used to absorb the excess B1 1

B1 cao must (cannot decipher copy properly)

(c) $\begin{bmatrix} 15 & & \\ 1 & 11 & 0 \\ & & 17 \end{bmatrix}$ B1 1

B1 cao total of five numbers

(d) Shadow costs $S_A = 0$ $S_B = -1$ $S_C = -1$

$D_1 = 62$ $D_2 = 49$ $D_3 = 1$

Improvement indices $I_{A2} = 47 - 0 - 49 = -2$*

$I_{A3} = 0 - 0 - 1 = -1$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 49 = 10$

	1 (62)	2 (49)	3 (1)
(0) A	15–θ	θ	
(–1) B	1+θ	11–θ	0
(–1) C			17

M1A1A1ft 3

Entering A2, exiting B2, $\theta = 0$

Shadow costs $S_A = 0$ $S_B = -1$ $S_C = -1$

$D_1 = 62$ $D_2 = 47$ $D_3 = 1$

Improvement indices $I_{A3} = 0 - 0 - 1 = -1$*

$I_{B2} = 48 + 1 - 47 = 2$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 47 = 12$

	1 (62)	2 (47)	3 (1)
(0) A	4–θ	11	θ
(–1) B	12+θ		0–θ
(–1) C			17

M1A1A1ft 3

Entering A3, exiting B3, $\theta = 0$

	1 (62)	2 (47)	3 (1)
(0) A	4	11	θ
(–1) B	12		
(–1) C			17

Shadow costs	$S_A = 0$	$S_B = -1$	$S_C = 0$		
	$D_1 = 62$	$D_2 = 47$	$D_3 = 0$	M1 A1	
Improvement indices	$I_{B2} = 48 + 1 - 47 = 2$				
	$I_{B3} = 0 + 1 - 0 = 1$				
	$I_{C1} = 68 - 0 - 62 = 6$ B1				
	$I_{C2} = 58 - 0 - 47 = 11$				
∴ Optimal					
Cost 1497 units				B1	4

[14]

9. Let x_{ij} be the number of units transported from i to j , in 1000 litres where $i \in \{F, G, H\}$ and $j \in \{S, T, U\}$ — B2, 1, 0 — 2

Minimise $C = 23\,x_{fs} + 31x_{fft} + 46x_{fu} +$ — B1

$35\,x_{gs} + 38x_{\,gt} + 51\,x_{gu} +$ — B1 — 2

$41\,x_{hs} + 50x_{ht} + 63x_{hu}$

Unbalanced

Subject to

$x_{fs} + x_{ft} + x_{fu} \le 540$

$x_{gs} + x_{gt} + x_{gu} \le 789$ — M1

$x_{hs} + x_{ht} + x_{hu} \le 673$ — A1

$x_{fs} + x_{gs} + x_{hs} \le 257$ }

$x_{ft} + x_{gt} + x_{ht} \le 348$ } accept = here — A1 — 3

$x_{fu} + x_{gu} + x_{hu} \le 412$ }

$x_{ij} \ge 0$ — B1 — 1

Accepted introduction of a dummy demand methods.

[8]

10. (a) Adds zero for costs in third column — B1

Adds 14 as the demand value — B1 — 2

(b) The total supply is greater than the total demand — B2, 1, 0 — 2

(c) The solution would otherwise be degenerate B2 1

(d)

		10	15	0
		J	K	L
0	A		8	1
0	B			13
–6	C	9	3	

$I_{AJ} = 12 – 0 – 10 = 2$ M1 A1
$I_{BJ} = 8 – 0 – 10 = –2$ A1
$I_{BK} = 17 – 0 – 15 = 2$ A1
$I_{CL} = 0 + 6 – 0 = 6$ 4

	J	K	L
A		$8 – \theta$	$1 + \theta$
B	θ		$13 – \theta$
C	$9 – \theta$	$3 + \theta$	

$\theta = 8$
Entering square BJ M1
Exiting square AK A1ft 2

		8	13	0
		J	K	L
0	A			9
0	B	8		5
–4	C	1	11	

$I_{AJ} = 12 – 0 – 8 = 4$ M1 A1ft
$I_{AK} = 15 – 0 – 13 = 2$ A1ft
$I_{BK} = 17 – 0 – 13 = 4$ A1ft
$I_{CL} = 0 + 4 – 0 = 4$ A1 5

No negatives, so optimal

[16]

11. (a)

	D	E	F
A	20	4	
B		26	6
C			14

M1
A1 2

(b) $S_A = 0$ $S_B = –1$ $S_C = 7$ M1
$D_P = 21$ $D_E = 24$ $D_F = 18$ A1

$I_{13} = I_{AF} = 16 – 0 – 18 = –2$
$I_{21} = I_{BD} = 18 + 1 – 21 = –2$ M1
$I_{31} = I_{CD} = 15 – 7 – 21 = –13$ (*) A1ft
$I_{32} = I_{CE} = 19 – 7 – 24 = – 12$ A1ft 5

(c) eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-)$ $\theta = 14$ M1 A1ft

	D	E	F
A	6	18	
B		12	20
C	14		

A1ft A1

cost £1384

4

[11]

12. Let x_{ij} be <u>number</u> of units transported from i to j
where $i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 1
warehouse supermarket

<u>objective</u> minimise "C" $= 3x_{WJ} + 6x_{WK} + 3x_{WL} +$ B1
$5x_{XJ} + 8x_{XK} + 4x_{XL} +$ B1 2
$2x_{YJ} + 5x_{YK} + 7x_{YL}$

subject to $x_{WJ} + x_{WK} + x_{UL} = 34$ M1 A1
$x_{XJ} + x_{XK} + x_{XL} = 57$
$x_{YJ} + x_{YK} + x_{YL} = 25$
$x_{WJ} + x_{XY} + x_{YJ} = 20$ A1 3
$x_{WK} + x_{XK} + x_{YK} = 56$
$x_{WL} + x_{XL} + x_{YL} = 40$

$x_{ij} \geq 0 \quad \forall \quad i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 1

[7]

13. (a) Idea of many supply and demand points and many units to be moved. Costs are variable and dependent upon the supply and demand points, need to minimise costs. <u>Practical</u> costs proportional to number of units B2, 1, 0 2

(b) Supply = 120 Demand = 110 so not balanced B1 1

(c) Adds 0, 0, 0, 10 to column f M1 A1

	d	e	f
A	45		
B	5	30	
C		30	10

M1 A1

Cost 545 B1 ft 5

(d) $R_1 = 0$ $R_2 = -1$ $R_3 = -3$
$k_1 = 5$ $k_2 = 7$ $k_3 = 3$ M1 A1

$Ae = 3 - 0 - 7 = -4$
$Af = 0 - 0 - 3 = -3$ M1 A1 ft
$Bf = 0 + 1 - 3 = -2$ A1 ft 5
$Cd = 2 + 3 - 5 = 0$

(e) $Ae^+ \rightarrow Be^- \rightarrow Bd^+ \rightarrow Ad^-$ send 30 M1 A1 ft

	d	*e*	*f*
A	15	30	
B	35		
C		30	10

Cost 425

depM1
A1 ft
A1 5

[18]

14. (a) e.g.

	D	E	F
A	6		
B	0	5	
C		4	4

<u>or</u>

	D	E	F
A	6	0	
B		5	
C		4	4

M1 A1

<u>cost £470</u> A1 3

(b) $S_A = 0, S_B = 0, S_C = -10$
$D_D = 20, D_E = 30, D_F = 40$
$I_{AE} = 40 - 30 = 10$
$I_{AF} = 10 - 40 = -30$
$I_{BF} = 40 - 40 = 0$
$I_{CD} = 10 - 10 = 0$

Choose *AF* as entering route

$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow BE(-) \rightarrow BD(+) \rightarrow AD(-)$
Exiting route *CF* $\theta = 4$

	D	E	F
A	2		4
B	4	1	
C		8	

$S_A = 0, S_B = 0, S_C = -10$
$D_D = 20, D_E = 30, D_F = 10$
$I_{AE} = 10, I_{BF} = 30,$
$I_{CD} = 0, I_{CF} = 30$
$\therefore$ optimal, cost £350

$S_A = 0, S_B = -10, S_C = -20$
$D_D = 20, D_E = 40, D_F = 50$ M1 A1
$I_{AF} = 10 - 50 = -40$
$I_{BD} = 20 - 10 = 10$
$I_{BF} = 40 - 40 = 0$
$I_{CD} = 10 - 0 = 10$ M1 A1 4

$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow AE(-)$
Exiting route *AE* $\theta = 0$ M1 A1 ft

	D	E	F
A	6		0
B		5	
C		4	4

A1 3

$S_A = 0, S_B = 30, S_C = 20$
$D_D = 20, D_E = 0, D_F = 10$
$I_{AE} = 40, I_{BD} = -30,$
$I_{BF} = 20, I_{CD} = -30$ M1 A1 A1
$CD(+) \rightarrow AD(-) \rightarrow AF(+) \rightarrow CF(-)$
$\theta = 4$

	D	E	F
A	2		4
B		5	
C	4	4	

$S_A = 0, S_B = 0, S_C = -10$
$D_D = 20, D_E = 30, D_F = 10$
$I_{AE} = 10, I_{BD} = 0, I_{BF} = 30, I_{CF} = 30$
$\therefore$ optimal, cost £350 A1 7

[14]

15.

Warehouse Factory	*A*	*B*	*C*	Capacity
1	20	16	24	300
2	10	10	8	500
3	12	18	10	100
Demand	200	400	300	900

Let x_{1A} be the number of units sent on route (1, A)

.

.

.

x_{3C} be the number of units sent on route (3, C) (variables) B1

Let the cost be £z.

Minimise total cost = sum of products of number and unit cost

Min $z = 20x_{1A} + 16x_{1B} + 24x_{1C}$

$+ 10x_{2A} + 10x_{2B} + 8x_{2C}$

$+ 12x_{3A} + 18x_{3B} + 10x_{3C}$ (objective function) B1

Total capacity = total demand

So all constraints are equations

Factory
1: $x_{1A} + x_{1B} + x_{1C} = 300$
2: $x_{2A} + x_{2B} + x_{2C} = 500$ M1 A1
3: $x_{3A} + x_{3B} + x_{3C} = 100$

Warehouse
1: $x_{1A} + x_{2A} + x_{3A} = 200$
2: $x_{1B} + x_{2B} + x_{3B} = 400$ M1 A1
3: $x_{1C} + x_{2C} + x_{3C} = 300$

All decision variables non-negative as negative number of units is not sensible.

$x_{1A}, x_{1B}, x_{1C}, \ldots, x_{3C} \geq 0$

[7]

16. (a) North-west corner rule

	A	*B*	*C*	
P	20	15		35
Q		20	40	60
S			30	30
	20	35	70	125

M1 A1 A1 3

(b)

	K_1	K_2	K_3
R_1	(0) [3]	(0) [3]	[9]
R_2	[6]	(0) [7]	(0) [6]
R_3	[5]	[2]	(0) [8]

$R_1 + K_1 = 3$ $R_1 + K_2 = 3$

$R_2 + K_2 = 7$ $R_2 + K_3 = 6$

$R_3 + K_3 = 8$

Taking $R_1 = 0 \Rightarrow K_1 = 3, K_2 = 3, R_2 = 4, K_3 = 2, R_3 = 6$

(R_S and K_S) M1 A1 A1

$I_{13} = 9 - 0 - 2 = 7$

$I_{21} = 6 - 4 - 3 = -1$

$I_{31} = 5 - 6 - 3 = -4$

$I_{32} = 2 - 6 - 3 = -7$

Some negative coefficients so not optimal. M1 A1 5

(c)

20	15	
	$20 - \theta$	$40 + \theta$
	$+\theta$	$30 - \theta$

Take $\theta = 20$ to obtain (θ) A1

20	15	
		60
	20	10

A1 3

(d)

	K_1	K_2	K_3
R_1	(0) [3]	(0) [3]	[9]
R_2	[6]	[7]	(0) [6]
R_3	[5]	[2]	(0) [8]

$R_1 + K_1 = 3$, $R_1 + K_2 = 3$
$R_2 + K_3 = 6$
$R_3 + K_2 = 2$, $R_3 + K_3 = 8$
Taking $R_1 = 0 \Rightarrow K_1 = 3, K_2 = 3, R_3 = -1, K_3 = 9, R_2 = -3$ M1 A1
$I_{13} = 9 - 0 - 9 = 0$
$I_{21} = 6 - (-3) - 3 = 6$
$I_{22} = 7 - (-3) - 3 = 7$
$I_{31} = 5 - (-1) - 3 = 3$ M1 A1

All non-negative and so optimal. Cost of this transportation pattern is

$20 \times 3 + 15 \times 3 + 60 \times 6 + 20 \times 2 + 10 \times 8$

$= 60 + 45 + 360 + 40 + 80 = 585$ M1 A1 6

[17]

1. No Report available for this question.

2. No Report available for this question.

3. No Report available for this question.

4. This also proved an excellent discriminator and a challenging question for some. Part (a) was correctly completed by most candidates, although some included extra zeros. Many candidates got the first set of shadow costs and improvement indices correct although some stated 8 'improvement indices'. Many were then able to find the correct stepping stone route, although some tried to include two empty cells in their route.

It cannot be stressed enough that in this algorithm candidates must differentiate between a cell containing the number zero and an empty cell. In the first application of the stepping stone route either XA or YC must be empty and the other one must contain the number zero.

Many solutions were muddled and poorly structured and although there were many blank tables printed in the answer book many candidates persist in trying to fit a solution, improvement indices and a stepping stone route all into one table making it very difficult for examiners to decipher. Some candidates did not recalculate their shadow costs and 4 improvement indices each time.

5. Most candidates started this question well and found the correct stepping stone route; few stated the correct exiting cell, with ZC being a very common incorrect answer. Many wasted time unnecessarily calculating shadow costs and improvement indices to confirm ZA as the entering cell. As in other papers improved solutions were marred by leaving zero in the exit cell. Those who found the correct (5 term) solution generally went on to find the correct shadow costs and improvement indices and the second improved solution, although many did not state the entering and exiting cells. Some candidates wasted time listing the stepping stone route rather than drawing it. Ample blank tables are provided in the answer book, but some candidates cram three or four sets of information into one table, often putting improved solution, improvement indices, new stepping stone route and costs all on one diagram.

6. This was a good early question for the candidates, and a good source of marks for most. Almost all the candidates gained some credit in part (a) for saying that supply was not equal to demand, but fewer stated that supply was greater than demand. Parts (b) and (c) were well done. Most found the correct first set of 5 shadow costs and 2 improvement indices, although some, often those who used a table to display them, included extra (zero) improvement indices. Although there were plenty of tables printed a few candidates tired to show the initial solution, improvement indices and stepping stone route all on just one diagram, making it very tricky to read. Most candidates found the correct stepping stone route and improved solution, although once again an extraneous zero sometimes appeared in the emptying square. A few candidates failed to recalculate shadow costs for the improved solution and some did not evaluate the new improvement indices. Most were able to find the correct cost.

7. Although this question was generally done quite well, a significant number of candidates made some errors in their working. Part (b) was often poorly done with many candidates just stating that supply did not equal demand. In part (c) there were slips in the calculating of shadow costs and improvement indices. There were a number of candidates who stated 9 improvement indices, candidates should appreciate the difference between a cell being ‘empty’ and having a zero entry. (A very few alarmingly included negative numbers in their NW corner method, which has not been seen before.) A significant number of candidates made errors on the stepping stone route, some did not use the most negative improvement index, some choose an invalid route with 2 empty cells others left a 0 in cell 3D. Some candidates wasted time, applying the algorithm twice here. Part (e) was generally well done, although a small number of candidates failed to calculate new shadow costs. Many candidates did not state their cost to 2dp, as is expected in money calculations.

8. This question caused problems for most candidates. Many candidates failed to give an adequate definition for degeneracy, with some referring to the “magic number” (m + n – 1), but did not define m and n. Others tried to describe the physical set of circumstances leading to degeneracy, but omitted part of the definition. Most candidates failed to correctly give the reason for a dummy, with the most common error being the statement that supply ≠ demand, rather than supply > demand. A significant minority stated that the number of suppliers did not equal the number of demand points. Most candidates gave the correct initial table, but many candidates then made errors. Common initial errors included miscalculating shadow costs or improvement indices, stating an incorrect stepping stone route or not giving the correct next solution. Those candidates who successfully completed the first iteration then ran into further problems, as many were confused by the zero in the dummy column. Some candidates did not calculate further shadow costs, believing that they had reached an optimum solution, others did not realise that their stepping stone route could have a � value of 0. A number of candidates just chose to move the 0 with no justification. Further errors then occurred with candidates not calculating their final improvement indices, or failing to draw a conclusion and calculating the cost incorrectly.

9. Some candidates did not clearly define their decision variables, X_{ij} must be defined as a **number** and many omitted the units of 1000 litres. Most stated the objective and the objective function correctly. The unbalanced problem caused difficulty for some, although others handled it very well – either by adding a dummy or by using inequalities to describe the constraints. Many poor answers were seen with equalities and once more the 1000 litre units caused difficulty for some. Most remembered the no-negativity constraints.

10. The vast majority of the candidates added the three zeros in part (a) and most added the 14. Part (b) caused some confusion, some felt that if there are n supply points then there must be n demand points too, most made it clear that the total supply was greater than the total demand but some wrote the weaker statement that the supply did not equal the demand. Most candidates were able to describe a degenerate solution in part (c). Some disappointing work was seen in part (d), some candidates had difficulty calculating the shadow costs and improvement indices, with negative signs casing most of the problems. Some did not find a valid stepping-stone route – using two empty squares or not using a negative improvement index as the entering square. Some did not introduce an exiting square – leaving a residual zero and thus creating an invalid solution.

11. Most candidates answered this very well, with only minor errors being made. These minor errors included numerical slips in calculating shadow costs and improvement indices, using two empty cells in a stepping stone route, nit indicating the stepping stone route, including an extra zero in the improved solution and choosing an incorrect theta value.

12. Some very good and some very poor answers were seen to this question. Some candidates did not define their decision variables, or changed notation mid-way through the question. Notation generally was sometimes weak. Many candidates were able to list the objective function correctly, but the constraints were often poorly handled with multiplication of variables by costs a surprisingly common sight.

13. The majority of the candidates found this question a good source of marks with parts (b) to (e) usually successfully answered. Once again the definition in part (a) proved problematical for many candidates, although most were able to earn some credit. The most omitted key element being that of variable costs. The rest of the question was well-answered, but the more common errors were: failure to state the costs in (c) and (e), not using the most negative improvement index, using more than one empty square in their stepping stone cycle and having greater than five numbers in their tables – often leaving a residual zero.

14. Many candidates initially did not realise that the initial solution was degenerate, although many recovered later. A number of candidates stated that they should add a zero to cell, but then failed to act upon this. Of those who completed part (a) most went on to correctly find the shadow costs and the four improvement indices, although some had a number other than 5 non-empty cells at some stage. The stepping-stone method caused problems. Some candidates ignored the cell with the negative index; others used two empty cells in their route.

15. No Report available for this question.

16. No Report available for this question.